

X. *On the Elliptic Polarization of Light by Reflexion from Metallic Surfaces.*
By the Rev. BADEN POWELL, M.A., F.R.S., F.G.S., F.R.A.S., Savilian Professor
of Geometry in the University of Oxford.

Received April 24,—Read June 19, 1845.

IN a former paper, inserted in the Philosophical Transactions, 1843, Part I., I detailed observations on some phenomena of elliptic polarization by reflexion from certain metallic surfaces; but with reference only to one class of comparative results. From these I have been led to pursue the subject into other relations besides those at first contemplated; but, from various causes, have only been able at this interval to submit the results to the Royal Society as a sequel to my former observations.

The changes in the degree of ellipticity, investigated in my former paper, correspond to certain changes in the thickness of metallic *films*. If we now consider the case of reflexion from a *simple polished metallic surface*, and admit that in this case it may be supposed to take place by the penetration of the ray to a certain minute depth, or to some action of a thin transparent lamina of the metal, then, in like manner,—dependent on the law of metallic retardation,—the effect would vary with a difference in the *effective* thickness of the lamina, produced by changing the inclination of the incident ray; and that this is the case in general is well known, viz. that as the incidence is increased, the ellipticity increases up to a maximum, which occurs for most metals at an incidence between 70° and 80° , beyond which it decreases up to 90° .

The original researches of Sir D. BREWSTER*, to which we are indebted for the first investigation of these phenomena, afford a striking instance of the legitimate process of inductive inquiry in its first stage, in their total exemption from all reference to any physical theory. In these researches, besides the change in ellipticity with the incidence, there is also included the change in the virtual plane of polarization by metallic reflexion, which, though conjectured to be the same as that investigated by FRESNEL for transparent substances†, was only examined in detail at incidences at or near that for the maximum ellipticity, to which the author's object immediately restricted him. But for a series of metals *constant arcs* were accurately determined‡, which are the azimuths of polarization of the ray, restored to plane polarization after *two* reflexions from metal plates at the incidence for the maximum; while the ellipse, from which the appellation of “elliptic polarization” was derived, is a purely empirical representation of the varying arcs of incidence, considered as

* Philosophical Transactions, 1830, Part II.

† Ibid. p. 292.

‡ Ibid. p. 294.

radii, for the second reflexion to restore plane polarization, at all azimuths of the plane of second reflexion to the first.

Though the subject of metallic reflexion is still in a condition of great obscurity, as to the mechanical causes to which its peculiar character is referrible, yet the application of the undulatory theory at least enables us to trace and connect some of its laws, and in the attempt to pursue such an application to some further relations, the nature of my researches may be briefly explained as follows :—

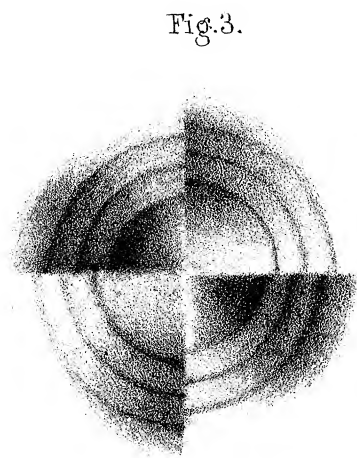
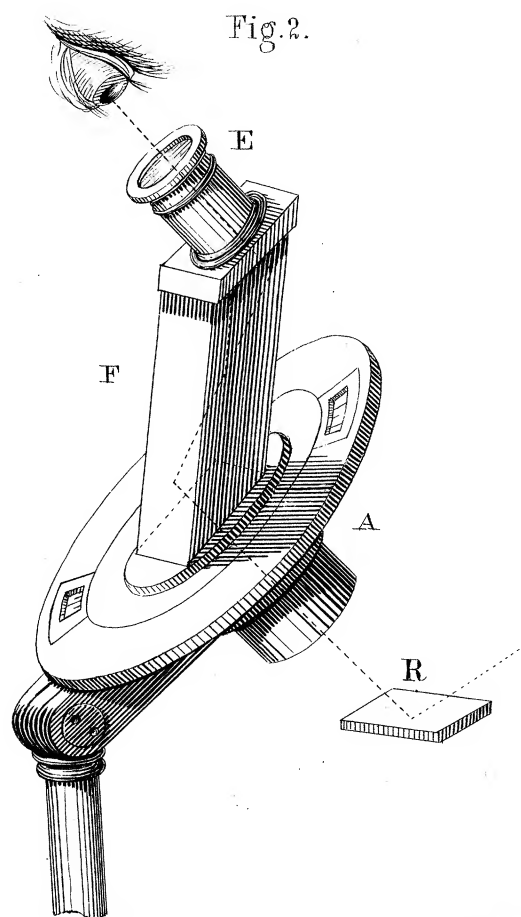
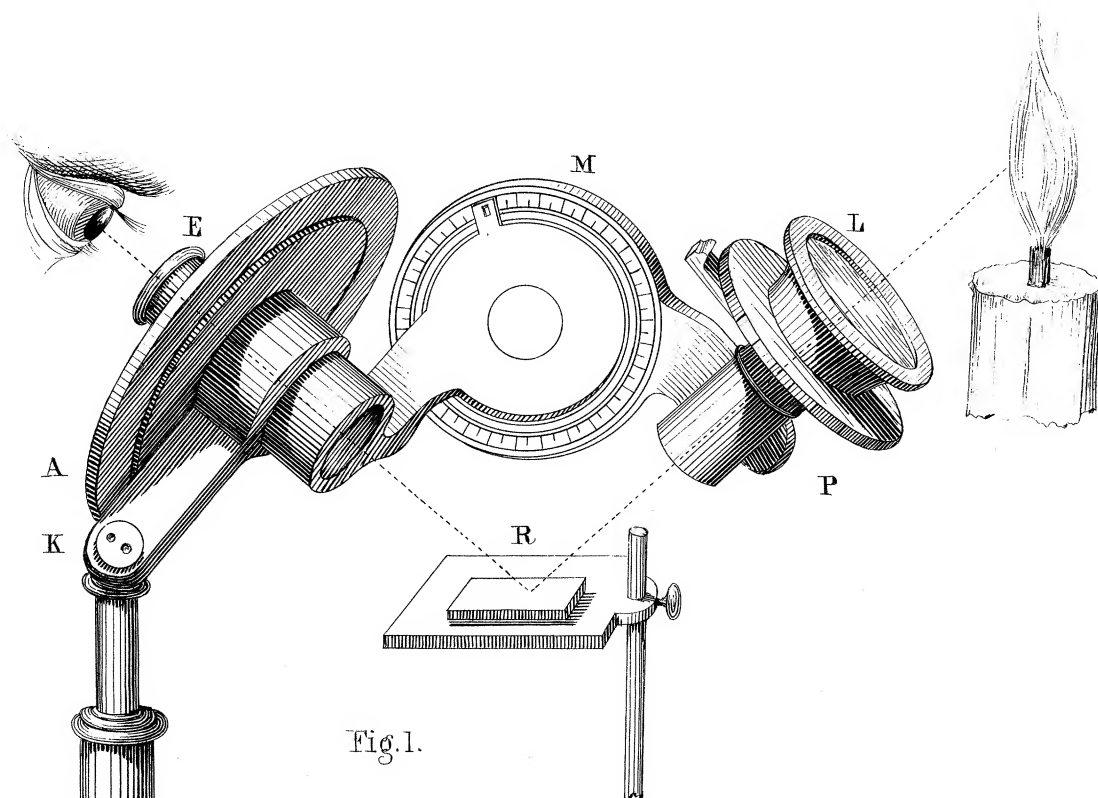
1. So far as the objects of my former inquiry were concerned, it sufficed to take the formula there employed for the polarized rings in the simplified form resulting from supposing a common coefficient to the two component vibrations; the plane of original polarization inclined 45° to that of reflexion, and 90° to that of analyzation. With reference however to some of the facts connected with observations at different incidences and azimuths of the polarizer, as well as on other grounds, it seemed desirable to generalize that formula by removing the above-mentioned restrictions; and I have accordingly here given an expression for the rings in elliptic light of all degrees with general coefficients, and for all positions of the polarizer and analyzer; which, though without difficulty deducible, has not, as far as I am aware, been stated by any writer.

2. With respect to the general character of *the rings*, the slightest observation shows that the distinction between the dark and bright centred systems in *plane* polarized light, though *modified*, is not *lost*, in the *lower* degrees of *ellipticity*; it disappears only when the light becomes perfectly circular; when the distinction is only seen in the changed direction of dislocation.

When the plane of analyzation is inclined 45° between the rectangular directions, and generally in intermediate positions, the whole appearance is, as it were, *distorted*; the dark arcs nearest the centre are situated towards *one end* of the quadrants, instead of being in the middle; and in the succeeding rings, though less strongly marked, there is an apparent increase of intensity towards the same *end* of the quadrant, owing to a general shade of darkness in the *ground* towards that side*. Of this appearance, though it must have been constantly seen, as far as I know, no explanation has been published. In *circular* polarization it does not occur. In plane polarized rings the analogous case is that of the well-known system of eight dislocated sectors; which in ellipticity of lower degrees is combined with, and passes into, that just described. All this is expressed by my formula.

3. The restoration of elliptic to plane polarized light by means of FRESNEL's rhomb, and the determination of the ellipticity by the azimuth of the rhomb, though an obvious process, yet has not, as far as I know, been pursued for any series of metals. Such a set of observations I have accordingly made at the incidence for the maximum ellipticity, for a considerable range of metals, some metallic ores, and other reflecting substances. Also in a few principal cases I have made similar observations at other incidences from 80° up to 30° , at which the ellipticity disappears.

* See Plate II. fig. 3.



4. In these cases the metallic reflexion performs the part of the first rhomb in FRESNEL's experiment with two. If I rightly interpret Sir D. BREWSTER's process for a similar restoration, before referred to, and if the two metal plates are analogous to the two rhombs, in the *change of plane*, which he so accurately determined, after the second reflexion, at the maximum incidence, we may infer a correspondence with that produced in the rhomb; and that it is *equal to twice the azimuth of the rhomb*. And in fact I find the results from two such different methods agree very closely for all the pure metals; though for certain ores, very low in the scale, there are some discrepancies.

There is also a close agreement between these results and the azimuths of the plane of the ray restored to plane polarization by the action of a crystallized plate, as given by the same author for some of the principal metals*, which confirms the same inference.

5. The *changes of plane* for successive incidences are in general of a nature *analogous* to those in the reflexion from transparent bodies, but not the *same*.

At the incidence for the maximum, if the plane of the polarizer be first adjusted to give, *e. g.* the dark system, then on changing its plane to 45° , the analyzer, in order to restore what is analogous to the same, that is, the *darkest* system, must coincide with the plane of incidence, or its azimuth is 0° , in exact agreement with what obtains both by theory and observation in the reflexion from transparent bodies.

At *greater* incidences, to restore the same system, the analyzer must be moved through increasing arcs on *one* side of the plane of incidence, and at *lesser* incidences on the *other*, as for transparent bodies.

Following up these latter, as the incidence is successively diminished, the azimuth of the analyzer changes in a manner obviously *different* from that which obtains for transparent bodies, though of the same *general* character: and the increase is slightly different for different metals; but in all cases it approaches 45° as the incidence approaches the perpendicular.

In these cases the azimuth of the polarizer remains at 45° . If it be changed, that of the analyzer preserves a constant relation.

Though these observations are of an obvious nature, yet, as far as I know, none of the kind have been published for any series of metals. I have accordingly given such a set for various reflecting substances.

But though apparently simple in principle, the process is troublesome in practice, and affected by various causes of uncertainty and difficulty. Some apparent anomalies, especially, which caused the arcs at small incidences to appear to increase beyond 45° , for a long time caused me much perplexity.

The results however here offered, having been obtained with the use of every precaution, and being the means of a great number of repetitions, I trust, on the whole, may not be useless as a first attempt to determine these changes by direct observation.

6. The ellipticity at different incidences cannot be deduced (except at the max-

* Philosophical Transactions, 1830, Part II. p. 311.

imum) from the rhomb observations without a knowledge of the change of plane: employing these latter data in combination with the former (3.), I have further estimated the ellipticity at different incidences for four principal metals.

7. For the application of the *undulatory theory* to these phenomena, we ought to be able to assign the law of metallic retardation, but this has not yet been done. The theory as here given indicates *the conditions of the maximum*, and shows in general *a change*, but not its *amount*.

Professor MACCULLAGH has however proposed, in accordance with a remarkable mathematical analogy, certain modifications of FRESNEL'S formula, which he has reduced to calculation in the case of *steel*.

My *theoretical* formula gives rise to an expression for the change of plane, but involving undetermined functions of the retardation. *On deducing the corresponding terms from Professor MACCULLAGH'S data, and introducing them into my formula, I find it gives a very close representation of the observed results for steel.* Thus Professor MACCULLAGH'S empirical expression receives an additional confirmation in accordance with a direct deduction from the undulatory theory.

The rest of this paper is devoted to the details of the observations, and of the analytical investigation.

THEORETICAL INVESTIGATION.

(1.) The original vibration in a plane P being

$$a \sin \frac{2\pi}{\lambda} (vt - x),$$

in general on reflexion in a plane R inclined to P by an angle ξ (R' being the plane perpendicular to R) it is resolved into

$$a \cos \xi \sin \frac{2\pi}{\lambda} (vt - x) \dots \text{in R,}$$

$$a \sin \xi \sin \frac{2\pi}{\lambda} (vt - x) \dots \text{in R'.$$

(2.) But in the case of *metallic* reflexion, one of these components is accelerated in phase by a quantity ϱ , and at the same time for the greater generality, supposing the coefficients unequal, or changing a in the 2nd formula to b , and writing for brevity

$$\alpha = a \cos \xi, \quad \beta = b \sin \xi;$$

after metallic reflexion the component vibrations will be

$$\alpha \sin \left(\frac{2\pi}{\lambda} (vt - x) \right) = R,$$

$$\beta \sin \left(\frac{2\pi}{\lambda} (vt - x) + \varrho \right) = R'.$$

(3.) Here we may remark that these formulas give directly the equation to the elliptic vibrations, the ratio of whose axes is that of α to β , which vary at different incidences

as well as with changes given to ξ ; or a and b are also functions of the incidence, or of ϱ . When $\varrho = \frac{\pi}{2}$ if $\alpha = \beta$ the light is circular; this is never the case in any metal at the maximum, though in some the ratio approaches it. For some other value of ϱ we may have $\alpha = \beta$, but this does not give circular polarization. When $\varrho = 0$ the same formulas give the inclination of the plane of the rectilinear vibrations*.

(4.) On interposing a plate of crystal cut perpendicular to its axis, for any plane Q in the crystal passing through the axis, inclined to R by an angle ϕ , and to which Q' is perpendicular, we have the vibrations

$$\begin{aligned} R \text{ resolved into } & \begin{cases} R \sin \phi \text{ in } Q' \\ R \cos \phi \text{ in } Q. \end{cases} \\ R' \text{ resolved into } & \begin{cases} R' \cos \phi \text{ in } Q' \\ R' \sin \phi \text{ in } Q. \end{cases} \end{aligned}$$

(5.) And on the general principle of resolution (observing that the resolved parts in one of the planes will be opposed), we have

$$\begin{aligned} R' \sin \phi + R \cos \phi &= Q \\ R' \cos \phi - R \sin \phi &= Q'. \end{aligned}$$

The vibrations in Q' form the ordinary ray O , and those in Q the extraordinary E . But after emergence the vibrations in Q are all further accelerated by θ , or become

$$\begin{aligned} \alpha \sin \left(\frac{2\pi}{\lambda} (vt - x) + \theta \right), \\ \beta \sin \left(\frac{2\pi}{\lambda} (vt - x) + \varrho + \theta \right). \end{aligned}$$

(6.) Thus we have

$$\begin{aligned} \left. \begin{aligned} & \beta \sin \left(\frac{2\pi}{\lambda} (vt - x) + \varrho \right) \cos \phi \\ & - \alpha \sin \left(\frac{2\pi}{\lambda} (vt - x) \right) \sin \phi \end{aligned} \right\} = O \dots \text{ in } Q'. \\ \left. \begin{aligned} & \beta \sin \left(\frac{2\pi}{\lambda} (vt - x) + \varrho + \theta \right) \sin \phi \\ & + \alpha \sin \left(\frac{2\pi}{\lambda} (vt - x) + \theta \right) \cos \phi \end{aligned} \right\} = E \dots \text{ in } Q. \end{aligned}$$

(7.) Now if the analyzer be applied with the plane of analyzation A , inclined to R by an angle χ , we shall have the angle $AQ = \psi = \chi - \phi$, and $AP = \chi + \xi$.

The vibrations being again resolved in A and A' perpendicular to it, we find

$$\begin{aligned} O \text{ resolved into } & \begin{cases} O \sin \psi \text{ in } A' \\ O \cos \psi \text{ in } A. \end{cases} \\ E \text{ resolved into } & \begin{cases} E \cos \psi \text{ in } A' \\ E \sin \psi \text{ in } A. \end{cases} \end{aligned}$$

After analyzation the parts transmitted are those only in A , or

$$O \cos \psi - E \sin \psi.$$

* See my Treatise on the Undulatory Theory, &c., p. 12.

(8.) On substituting the values of O and E, and arranging the terms, this is reducible to the form

$$H \sin\left(\frac{2\pi}{\lambda}(vt-x)\right) + K \cos\left(\frac{2\pi}{\lambda}(vt-x)\right),$$

where

$$\left. \begin{aligned} & \beta \cos \phi \cos \psi \cos \epsilon - \alpha \sin \phi \cos \psi \\ & - \beta \sin \phi \sin \psi \cos \epsilon \cos \theta \\ & + \beta \sin \phi \sin \psi \sin \epsilon \sin \theta \\ & - \alpha \cos \phi \sin \psi \cos \theta \end{aligned} \right\} = H,$$

and

$$\left. \begin{aligned} & \beta \cos \phi \cos \psi \sin \epsilon \\ & - \beta \sin \phi \sin \psi \sin \epsilon \cos \theta \\ & - \beta \sin \phi \sin \psi \cos \epsilon \sin \theta \\ & - \alpha \cos \phi \sin \psi \sin \theta \end{aligned} \right\} = K.$$

Then, since the intensity I at any part of the image is expressed by

$$I = H^2 + K^2,$$

(9.) Squaring these quantities H and K, and taking the sum, after reduction, we ultimately find for any value of χ , or position of the analyzer,

$$I = \left\{ \begin{aligned} & (\beta^2 \cos^2 \phi + \alpha^2 \sin^2 \phi) \cos^2 \psi \\ & + (\beta^2 \sin^2 \phi + \alpha^2 \cos^2 \phi) \sin^2 \psi \\ & - \alpha\beta \sin 2\phi \cos 2\psi \cos \epsilon \\ & - \alpha\beta \cos 2\phi \sin 2\psi \cos \epsilon \cos \theta \\ & - \alpha\beta \sin 2\psi \sin \epsilon \sin \theta. \end{aligned} \right.$$

(10.) Or, in order to see the consequences of changing the position of the analyzer, or the arc χ , we must introduce it by substituting for ϕ its value $\phi = \chi - \psi$,

$$I = \left\{ \begin{aligned} & (\alpha^2 \sin^2 (\chi - \psi) + \beta^2 \cos^2 (\chi - \psi)) \cos^2 \psi \\ & + (\alpha^2 \cos^2 (\chi - \psi) + \beta^2 \sin^2 (\chi - \psi)) \sin^2 \psi \\ & - \alpha\beta \cos 2\psi \sin 2(\chi - \psi) \cos \epsilon \\ & - \alpha\beta \sin 2\psi \cos 2(\chi - \psi) \cos \epsilon \cos \theta \\ & - \alpha\beta \sin 2\psi \sin \epsilon \sin \theta. \end{aligned} \right.$$

(11.) On expanding and reducing this becomes—

$$I = \left\{ \begin{aligned} & (\alpha^2 \sin^2 \chi + \beta^2 \cos^2 \chi) \cos^4 \psi \quad . \quad . \quad . \quad 1. \\ & + (\alpha^2 \sin^2 \chi + \beta^2 \cos^2 \chi) \sin^4 \psi \quad . \quad . \quad . \quad 2. \\ & + 2(\alpha^2 \cos^2 \chi + \beta^2 \sin^2 \chi) \sin^2 2\psi \quad . \quad . \quad . \quad 3. \\ & - \frac{1}{2} (\alpha^2 - \beta^2) \sin 2\chi \sin 2\psi \cos 2\psi \quad . \quad . \quad . \quad 4. \\ & - \alpha\beta \sin 2\chi \cos^2 2\psi \cos \epsilon \quad . \quad . \quad . \quad 5. \\ & + \alpha\beta \cos 2\chi \sin 2\psi \cos 2\psi \cos \epsilon \quad . \quad . \quad . \quad 6. \\ & - \alpha\beta \cos 2\chi \sin 2\psi \cos 2\psi \cos \epsilon \cos \theta \quad . \quad . \quad 7. \\ & - \alpha\beta \sin 2\chi \sin^2 2\psi \cos \epsilon \cos \theta \quad . \quad . \quad . \quad 8. \\ & - \alpha\beta \sin 2\psi \sin \epsilon \sin \theta \quad . \quad . \quad . \quad . \quad 9. \end{aligned} \right.$$

(12.) This formula is general for all positions of the polarizer and analyzer, and for light of all degrees of ellipticity. If we had taken (as in my former paper) $\psi = (45 - \phi)$, and at the same time supposed $\alpha = \beta = 1$ (which is equivalent to the assumption then made for simplification), the formula (9.) would become at once the same as in that paper.

The first terms independent of θ express the intensity of the *ground* on which, as it were, the rings are formed: those involving ϵ vary with the incidence or degree of ellipticity: those involving θ give the rings; the last only being retained when the ellipticity is a maximum, and disappearing for plane polarization at the incidence 0.

From these formulas we at once trace all the well-known phenomena of the rings in plane and elliptic light, by following the changes in formula (9.) on advancing into the adjacent quadrants, or supposing ψ to become $\psi + 90$, and by consequence changing also ϕ into $\phi + 90$; which gives a change of sign in the terms involving $\sin 2\psi$, $\cos 2\psi$, $\sin 2\phi$, $\cos 2\phi$.

Again, with regard to changes in the analyzer, it is evident that whatever be the value of χ if we increase it by 90° , we shall have to substitute in (10.) $\cos(\chi - \psi)$ for $\sin(\chi - \psi)$ and *vice versa*; as well as $-\sin 2(\chi - \psi)$ for $\sin 2(\chi - \psi)$, and $-\cos 2(\chi - \psi)$ for $\cos 2(\chi - \psi)$, or, on the whole, the expression is obviously complementary, except the last term.

(13.) In any case on giving successive values to ψ round the rings, at those points where the sines or cosines vanish, the disappearance of any term when accompanied by a change of sign indicates (so far as that term affects the total intensity) a change from dark to bright at that point, or a complementary character in the adjacent portions of the image; when without a change of sign it indicates a simple maximum or minimum of light, in the rings or in the ground, according as the terms affected involve θ or not; and for light of different ellipticities according to the value of ϵ .

Thus for the values of χ in general, in (11.)—

At $\psi = 0$, or 90° ,

terms 1, 2, 3, for the ground	}	disappear without change of sign.
8, for the rings		
4, 6, for the ground	}	disappear with change of sign.
7, 9, for the rings		

At $\psi = 45$,

5, for the ground,	disappears without change of sign.
4, 6, for the ground	}
7, for the rings	
	} disappear with change of sign.

In *general* then there are dislocations of the *rings*, with more or less complete complementary changes in the *ground* both at the quadrants and half-quadrants.

At the maximum ellipticity, since terms 5, 6, 7, 8 disappear, there are dislocations only at $\psi = 0$, with a change in the *ground* at $\psi = 45$, or a gradual decrease of brightness from one *end* of the quadrant to the other, which is exactly the appearance observed. In plane polarized light we have the system of eight dislocated sectors,

intermediate between the dark and bright systems. In the lower degrees of ellipticity this is modified by, and passes into, that just described.

(14.) We may illustrate the application of the formula by one or two particular cases :—

1st. If we suppose at some incidence $\alpha = \beta$ while $\epsilon < \frac{\pi}{2}$, then $\chi = 45^\circ$ will give branches with dislocation at $\psi = 0^\circ$, but none at $\psi = 45^\circ$; that is the nearest approach to the dark system. This agrees with formula (15.), where in this case I is a minimum when $\chi = 45^\circ$, also with (18.).

2ndly. On the same supposition $\chi = 0$ will give complementary changes both in the ground and in the rings at $\psi = 0$, and similar changes, though less conspicuous, at $\psi = 45^\circ$; that is the intermediate system.

3rdly. For the general values of α and β at the maximum $\cos \epsilon = 0$; and $\chi = 0$ gives branches with dislocation at $\psi = 0$, but no change at $\psi = 45^\circ$; or the darkest system. But $\chi = 45^\circ$ will give (since $\alpha > \beta$) a complementary change in the ground at $\psi = 45^\circ$, and branches with dislocation at $\psi = 0$; or the distorted system.

But if in this case $\alpha = \beta$, or the polarization be *circular*, the term (4.) disappears, and there is no distorted system in any position of χ .

Observation shows this to be the case in perfectly circular light, and very nearly so in the higher degrees of ellipticity.

(15.) For the branches, when $\psi = 0$, for χ in general we have

$$I = \alpha^2 \sin^2 \chi + \beta^2 \cos^2 \chi - \alpha\beta \sin 2\chi \cos \epsilon.$$

Hence, on making successively $\cos \epsilon = 1$, $\cos \epsilon = 0$, $\cos \epsilon = -1$, &c., it is obvious that the intensity of the branches for the maximum ellipticity would be a mean between that in the dark and bright systems of plane polarized light if α and β were the same in the respective cases, which we shall see is the case; at all events, this relation of the intensities agrees with observation as far as the eye can judge.

(16.) Again, for the maximum ellipticity,

$$I = \beta^2 + (\alpha^2 - \beta^2) \sin 2\chi,$$

which can never be $= 0$; or the branches are never absolutely dark; but it is evidently a minimum when $\chi = 0$, and a maximum when $\chi = 90^\circ$, in which cases respectively $I = \beta^2$, or $I = \alpha^2$. If the polarization were circular these values would be equal, or the brightness the same in all positions of the analyzer.

(17.) For incidence 0° , the expression (15.) being made $= 0$, or,

$$I = \alpha^2 \sin^2 \chi + \beta^2 \cos^2 \chi - 2\alpha\beta \sin \chi \cos \chi = 0,$$

we have for the position of the analyzer for absolutely dark branches, as in (3.),

$$\alpha \sin \chi - \beta \cos \chi = 0,$$

or

$$\tan \chi = \frac{\beta}{\alpha} \tan \xi.$$

But observation shows in this case that $\tan \chi = 45^\circ$ when $\xi = 45^\circ$ in all instances, hence for incidence 0° $a = b$.

(18.) More generally, differentiating (15.) in respect of χ , we find for any given value of ξ , that is of α and β , the value of χ for the minimum, or darkest branches,

$$\tan 2\chi = -\frac{2\alpha\beta \cos \xi}{\alpha^2 - \beta^2},$$

or we might deduce directly

$$\tan \chi = -\frac{\alpha^2 - \beta^2}{2\alpha\beta \cos \xi} \pm \sqrt{\left(1 + \frac{(\alpha^2 - \beta^2)^2}{4\alpha^2\beta^2 \cos^2 \xi}\right)}.$$

Hence we may make the same inferences as before.

And if when $\cos \xi = 0$ we have also $\alpha = \beta$, or *circular* vibrations, it is worth while to observe that there results

$$\tan 2\chi = \frac{0}{0}.$$

But in general for the change of plane, or of χ , this formula does not assign any precise values, since the form of the function of ξ involved in α and β is unknown.

(19.) In the absence of any *theoretical* law I have had recourse to the *empirical* modification of FRESNEL'S formula proposed by Professor MACCULLAGH*. And availing myself of the computed values which he has given, in the case of steel, for the coefficients of the component vibrations α' (assuming $\alpha' = \beta$ in my notation) and the retardations δ δ' (where $\delta - \delta' = \xi$ in my notation), I have deduced the values of the product $\alpha \alpha'$, of $\alpha^2 - \alpha'^2$, and of $\cos \xi$, and introducing these in formula (18.), I find the resulting values of χ in close agreement with those given by observation. The elements of this computation and its results are exhibited in Table IV.

APPARATUS.

The general principle of my apparatus will be apparent from the nature of the experiments, but it may not be superfluous (especially for those who may wish to pursue similar observations) to annex an outline of the construction, which, after trial of various forms, I have found best to unite the requisite conditions for the purpose in view; while it is readily convertible into an ordinary polariscope; besides admitting of the addition or substitution of other parts, when required, for different objects.

The accompanying Plate II. fig. 1 gives a general view of the arrangement and will sufficiently explain itself: the polarizing part (P) contains a NICOL prism which can be turned in azimuth, measured by a graduated circle; a condensing lens (L) is also attached to it.

The analyzing part (A) contains in the eye-piece (E) a lens, a calc-spar, and a tourmaline; and bears a graduated circle, on which the azimuth of analyzation is read off.

The metal under examination is placed on the support (R), which can be raised or

* Reports, Royal Irish Academy, October 1836.

lowered as required; and should be capable of a slight inclination for better adjustment.

The middle part or hinge (M) is surrounded by a graduated vertical circle, by which the parts (P) and (A) can be set at any angle to each other: they are attached to it by projecting arms: the whole is moved by the joint (K) till the reflected light from (R) comes distinctly to the eye through (E), when (it is easily seen) the angle of incidence will be *half* that measured on (M).

The zero of (M) is found by adjusting (P) and (A) with their axes in one line (or reducing the instrument to an ordinary polariscope). The hinge should be capable of being firmly clamped.

In observations at very great incidences it will be easily seen to be necessary to have the arms carrying (P) and (A) so fixed, that when the axes of the tubes are brought into one line there shall be a considerable space between their inner ends: there should be also a slight motion about the axis of the arm for adjustment in (P).

In observations at very small incidences, it is most convenient to throw the light on to (P) by means of a small mirror placed in a proper position.

Fig. 2 represents the mode of applying the FRESNEL rhomb (F) to the analyzing part, the rest of the arrangement remaining the same. The eye-piece (E) is removed to a cap on the top of the rhomb; while the bottom of the rhomb is attached to a short tube, which fits into the aperture before occupied by the eye-piece.

Fig. 3 represents the appearance of the rings in elliptic light when the analyzer is at 45° between the dark and light systems.

OBSERVATIONS.

The determination of the precise position of the analyzer at which the maximum or minimum brightness is attained, is from the nature of the case open to considerable uncertainty. But the intervention of the *rings*, instead of using the simple analyzer, affords some aid in this respect, since the change in the nature and form of the rings offers a more ready guide to the eye; especially in elliptic light, where there is never a total evanescence. Still the undefined nature of the object observed precludes minute numerical accuracy, and the results in the following tables must be regarded as no more than approximations, though derived from the means of a great number of repetitions. In taking these means, I have omitted fractions of a degree as bearing an appearance of accuracy quite illusory.

The greatest care is necessary in the adjustments, especially of the polarizer. They were remade for each set of observations. That for the polarizer to obtain the zero or coincidence with the plane of reflexion, was found by varying the azimuth till the light was perfectly restored to plane polarization; the analyzer being in the position for dark branches, the zero of its circle was determined at the same time.

In the observations at different incidences, in order to secure the constancy of its inclination to the plane of incidence, the polarizer was adjusted to zero before obser-

vation at each incidence. For some of the ores, &c. which reflect but little light, the arcs must be taken as mere estimations. I have found some peculiar discordances between different sets of observations, especially in silver and copper: those given are the means.

The observations here annexed in a tabular form consist of,—

1. Those with FRESNEL'S rhomb for ascertaining from the azimuth (γ) for the restoration of plane polarization, the ellipticity at the incidence for the maximum; or giving

$$\epsilon = \left(\frac{\beta}{\alpha}\right)_m = \tan \gamma_m.$$

These results for the principal metals and other substances are seen in Table I., columns 3 and 4.

Again, in the observations with the rhomb at azimuth γ , the plane of the emergent ray is inclined by 2γ to the original plane. In Sir D. BREWSTER'S experiments with two metal plates it is inclined $45 + \phi$. The results are here compared in columns 1 and 2, and sufficiently show that we have

$$\gamma = \frac{45 + \phi}{2}$$

for all the pure metals.

Also we may remark, that Sir D. BREWSTER'S empirical ellipse has the ratio of its axes determined by $\sin 2\phi$, and is therefore different from the undulatory.

The same author also gives* the azimuths of the plane of the ray restored to plane polarization by a crystallized plate for several metals, as follows:—

Pure silver	42°	Speculum	32°
Copper	36° 30'	Steel	30° 30'
Mercury	35°	Lead	26°
Platinum	34°	Galena	17° 30'

In each instance it will be seen (on comparison with Table I.), that as nearly as possible the arc $= \frac{45 + \phi}{2}$; with the exception of galena.

2. Observations for the values of χ at successive incidences, while ξ remains at 45° .

These series of arcs are given in Table II., column 1, for the two metals of greatest and least maximum ellipticity and two intermediate; and in Table III. for a number of others.

3. Observations for the ellipticity at different incidences.

In general the ellipticity is measured by the inclination of the rhomb (γ) to the plane of previous polarization.

For the maximum ellipticity, since $\chi=0$, it follows that γ is correctly measured from the plane of reflexion. But at *other* incidences γ must be corrected by the change of plane of polarization due to that incidence, or we must take $(\gamma - \chi)$.

In Table II. (columns 2 and 3) are given the observed values of γ for four principal

* Philosophical Transactions, 1830, Part II. p. 311.

metals at different incidences, and the corresponding ellipticities (ε) or $\tan(\gamma - \chi)$. These results give on the whole a view of the changes of ellipticity agreeing with the general appearance of the rings; though in copper the diminution appears too rapid.

TABLE I.

Metal.	Values of ϕ (BREWSTER).	$\frac{45+\phi}{2}$.	Azimuth of rhomb = γ .	Ellipticity $\varepsilon = \tan \gamma$.
Silver, pure	$39^{\circ} 48'$	$42^{\circ} 24'$	42°	$\cdot 932$
Silver, common	36	40 30	40	$\cdot 839$
Gold, pure	35	40		
Gold, common	33	39	40	$\cdot 839$
Brass.	32	38 30	38	$\cdot 781$
Copper	29	37	38	$\cdot 781$
Mercury	26	35 30	35	$\cdot 700$
Platinum	22	33 30	34	$\cdot 674$
Palladium	34	$\cdot 674$
Speculum	21	33	31	$\cdot 600$
Bismuth	21	33	34	$\cdot 674$
Zinc	19 10	32 5	32	$\cdot 624$
Steel	17	31	30	$\cdot 577$
Iron	29	$\cdot 554$
Antimony	16 15	30 37	31	$\cdot 600$
Lead	11	28	28	$\cdot 531$
Tinned plate.	32	$\cdot 624$
Plated copper	32	$\cdot 624$
Iron pyrites	14	29 30	29	$\cdot 554$
Galena	2	23 30	24	$\cdot 466$
Specular iron ore.	0	22 30	24	$\cdot 466$
Tempered steel, yellow	20	$\cdot 363$
Tempered steel, blue.	15	$\cdot 267$
Plumbago.	15	$\cdot 267$
Decomposed glass	7	$\cdot 122$

TABLE II.

Pure silver ($\xi = 45^{\circ}$).					Copper ($\xi = 45^{\circ}$).			
Observed values of					Observed values of			
χ .	χ .	γ .	$\gamma - \chi$.	ε .	χ .	γ .	$\gamma - \chi$.	ε .
30	42	45	3	$\cdot 052$	44	45	1	$\cdot 017$
40	36	45	9	$\cdot 158$	40	45	2	$\cdot 034$
50	31	45	14	$\cdot 249$	23	44	11	$\cdot 194$
60	25	44	19	$\cdot 344$	24	43	19	$\cdot 344$
70	10	43	34	$\cdot 674$	10	29	29	$\cdot 554$
73) 0	42	42	0	$\cdot 932$	73) 0	38	38	$\cdot 781$
80	-25	45	20	$\cdot 363$	-21	40	19	$\cdot 344$
Steel ($\xi = 45^{\circ}$).					Lead ($\xi = 45^{\circ}$).			
Observed values of					Observed values of			
χ .	χ .	γ .	$\gamma - \chi$.	ε .	χ .	γ .	$\gamma - \chi$.	ε .
30	43	45	2	$\cdot 034$	45	45	0	$\cdot 0$
40	41	45	4	$\cdot 069$	42	45	3	$\cdot 052$
50	34	43	9	$\cdot 158$	30	43	13	$\cdot 230$
60	29	41	13	$\cdot 230$	24	42	18	$\cdot 324$
70	15	35	20	$\cdot 363$	8	33	25	$\cdot 466$
75) 0	30	30	0	$\cdot 577$	72) 0	28	28	$\cdot 531$
80	-19	34	15	$\cdot 267$	-14	35	21	$\cdot 383$

TABLE III.

i .	Observed values of χ . $\xi = 45^\circ$.							
	Gold.	Brass.	Mercury.	Platinum. Palladium.	Bismuth.	Zinc.	Antimony.	Speculum.
30	44	43	45	44	43	44	44	44
40	40	42	44	43	42	43	43	42
50	36	39	42	40	41	41	42	40
60	17	35	39	38	36	39	38	32
70	0	15	29	32	23	25	25	17
		72) 0	78) 0	78) 0	74) 0	77) 0	75) 0	76) 0
80	-30	-19	-7	-10	-25	-10	-20	-20

i .	Observed values of χ . $\xi = 45^\circ$.							
	Plated copper.	Tinned plate.	Tempered steel, yellow.	Tempered steel, blue.	Decom- posed glass.	Specular iron ore.	Galena.	Plumbago.
30	43	44	43	43	43	43	44	45
40	41	43	40	41	42	42	43	44
50	40	41	26	35	38	35	40	35
60	37	40	0	10	9	27	30	24
				62) 0	62) 0			
70	20	22	-25	-21	-35	17	15	10
	72) 0	77) 0	77) 0	72) 0	75) 0
80	-30	-15	-45	-44	-44	-10	-20	-15

TABLE IV.

Steel.					
i .	From MACCULLAGH'S data.			Values of χ .	
	$\alpha\alpha'$.	$\alpha^2 - \alpha'^2$.	$\epsilon = \delta - \delta'$.	Calculated by (18).	Observed.
0	•526	0•0	0	45	nearest } 45 0°
30	•522	•100	8	42 14	43
45	•408	•231	19	36 40	36
60	•474	•421	41	29 45	29
75	•451	•610	90	0	0
85	•682	•456	150	-28 5	-27
90	1•0	0•0	180	-45	nearest } -43 90°

POSTSCRIPT.

Received May 8, 1845.

Since the foregoing paper was communicated I have obtained some additional instances of elliptic polarization, which may be not uninteresting in connexion with those therein mentioned.

Besides plumbago (in which the proportion of metal present is at least very variable and small) I have now observed a slight, but quite unequivocal ellipticity in the reflexion from *China ink*, which is wholly non-metallic. It is at its maximum at an incidence of 62° , and its azimuth of restoration by the rhomb is about 5° .

I have examined also *chromate of lead*, which gives very small ellipticity, which is at a maximum at an incidence of 70° (agreeing well with the mean refractive index), and the azimuth of the rhomb is about 6° .